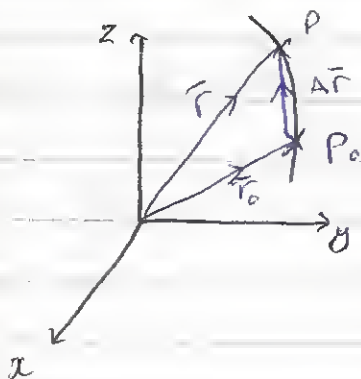


Alaa Eldin Tamer

Sec B

Curvilinear motion



$$\vec{D} = \vec{\Delta r} = \Delta x \hat{i} + \Delta y \hat{j} + \Delta z \hat{k}$$

$$\vec{D} = \vec{\Delta r} = \vec{r} - \vec{r}_0$$

$\vec{\Delta r} = \vec{D}$ = displacement

$$\langle \vec{v} \rangle = \lim_{\Delta t \rightarrow 0} \frac{\vec{\Delta r}}{\Delta t} \Rightarrow \vec{v} = \frac{d\vec{r}}{dt}$$

$$= \frac{dx}{dt} \hat{i} + \frac{dy}{dt} \hat{j} + \frac{dz}{dt} \hat{k}$$

$$= \dot{x} \hat{i} + \dot{y} \hat{j} + \dot{z} \hat{k}$$

$$= V_x \hat{i} + V_y \hat{j} + V_z \hat{k}$$

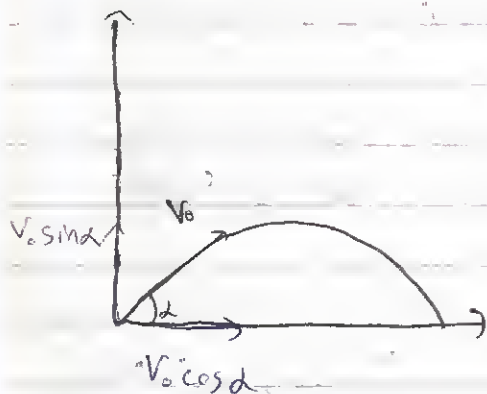
$$V = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}$$

$$\begin{aligned} \dot{x} &= \frac{dx}{dt} \\ \dot{y} &= \frac{dy}{dt} \\ \dot{z} &= \frac{dz}{dt} \end{aligned}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \ddot{x} \hat{i} + \ddot{y} \hat{j} + \ddot{z} \hat{k}$$

$$a = \sqrt{\ddot{x}^2 + \ddot{y}^2 + \ddot{z}^2}$$

projectile



$$\vec{a} = \ddot{x} \hat{i} + \ddot{y} \hat{j} \quad \ddot{x} = 0 \quad \ddot{y} = -g$$

$$\frac{dx}{dt} = 0 \quad \dot{x} = \text{constant}$$

$$\dot{x} = V_0 \cos \alpha$$

$$\frac{dx}{dt} = V_0 \cos \alpha$$

$$\int dx = \int V_0 \cos \alpha dt$$

$$x = V_0 \cos \alpha t$$

$$\ddot{y} = -g = \frac{dy}{dt} \quad \int dy = \int -g dt \quad \dot{y} = -gt + C$$

$$V_0 \sin \alpha = 0(t) + C$$

$$C = V_0 \sin \alpha \Rightarrow$$

$$\dot{y} = -gt + V_0 \sin \alpha$$

$$\dot{y} = \frac{dy}{dt} = V_0 \sin \alpha - gt$$

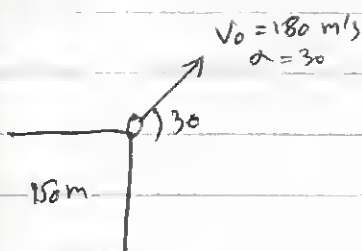
$$\int dy = \int V_0 \sin \alpha - gt \, dt$$

$$y = V_0 \sin \alpha t - \frac{gt^2}{2} + C$$

but $C=0$
(initial position is origin)

$$y = V_0 \sin \alpha t - \frac{gt^2}{2}$$

Ex 11.7



a) Find distance to position where particle hits the ground

$$y = V_0 \sin \alpha t - \frac{gt^2}{2}$$

$$-150 = 180 \sin 30 t - \frac{9.81}{2} t^2$$

$$t = 19.9$$

$$X = V_0 \cos \alpha t = 180 \cos 30 \times 19.9 = 3102 \text{ m}$$

b) Find velocity when it hits the ground

$$\dot{X} = V_0 \cos \alpha = 180 \cos 30 = 90\sqrt{3}$$

$$\dot{Y} = V_0 \sin \alpha - gt = 180 \sin 30 - (9.81 \times 19.9) = -10.5 \text{ m/s}$$

$$V = \sqrt{(90\sqrt{3})^2 + (10.5)^2} = 188 \text{ m/s}$$

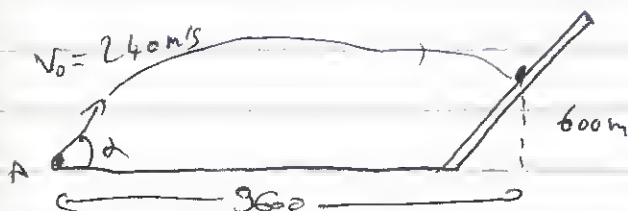
c) Find highest distance above ground

$$\dot{y} = V_0 \sin \alpha - gt = 0 \quad t = 9.17$$

$$y = -\frac{gt^2}{2} + V_0 \sin \alpha t$$

$$y = \frac{-9.81(9.17)^2}{2} + 180 \sin 30 \times 9.17 = 412.5$$

Ex 11.8



find α

$$X = V_0 \cos \alpha t \quad 3600 = 240 \cos \alpha t$$

$$t = \frac{15}{\cos \alpha}$$

$$y = -\frac{gt^2}{2} + V_0 \sin \alpha t$$

$$600 = -4.95 \left(\frac{15}{\cos \alpha} \right)^2 + 240 \times 15 \tan \alpha$$

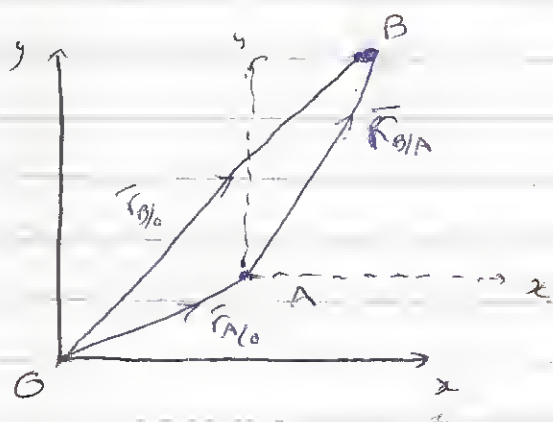
$$600 = -1104 (1 + \tan^2 \alpha) + 3600 \tan \alpha$$

$$-1104 \tan^2 \alpha + 3600 \tan \alpha - 1704 = 0$$

$$\tan \alpha = 2.69, 0.57$$

$$\alpha = 69.6, 29.7$$

Relative motion



$$\begin{aligned} \vec{r}_B &= \vec{r}_A + \vec{r}_{B/A} \\ \vec{r}_{B/A} &= \vec{r}_B - \vec{r}_A \\ \vec{v}_{B/A} &= \vec{v}_B - \vec{v}_A \\ \vec{a}_{B/A} &= \vec{a}_B - \vec{a}_A \end{aligned}$$

Ex 11.9 $v_A = 10 \text{ m/s}$ $r_A = 10 \times 5 = 50 \hat{i}$

$v_B = at + u = 1.2 \times 5 = 6 \text{ m/s}$

$v_B = -6 \hat{j}$

$y_B = y_0 - \frac{1}{2}at^2 = 35 - \frac{1}{2} \times 1.2 \times 5^2 = 20 \text{ m}$

$r_B = -20 \hat{j}$

$a_B = -1.2 \hat{j}$

$r_{B/A} = -20 \hat{j} - 50 \hat{i}$

$v_{B/A} = -6 \hat{j} - 10 \hat{i}$

$a_{B/A} = -1.2 \hat{j}$